Journal of the Egyptian Mathematical Society (2014) 22, 143–151

Egyptian Mathematical Society

Journal of the Egyptian Mathematical Society

www.etms-eg.org
www.elsevier.com/locate/joems

Peristatcally induced MHD slip flow in a porous medium due to a surface acoustic wavy wall

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Received 10 February 2013; revised 29 April 2013; accepted 5 May 2013
Available online 19 June 2013

KEYWORDS
Peristaltic transport
Boundary layer with peristalsis
Induced MHD Flow
Wavy walls
Porous medium

Abstract The influences of Hall current and slip condition on the MHD flow induced by sinusoidal peristaltic wavy wall in two dimensional viscous fluid through a porous medium for moderately large Reynolds number is considered on the basis of boundary layer theory in the case where the thickness of the boundary layer is larger than the amplitude of the wavy wall. Solutions are obtained in terms of a series expansion with respect to small amplitude by a regular perturbation method. Graphs of velocity components, both for the outer and inner flows for various values of the Reynolds number, slip parameter, Hall and magnetic parameters are drawn. The inner and outer solutions are matched by the matching process. An interesting application of the present results to mechanical engineering may be the possibility of the fluid transportation without an external pressure.

MATHEMATICAL SUBJECT CLASSIFICATION: 76D, 76S05, 76W, 76Z05

1. Introduction

The study of fluids flow induced by unsteady motion of a wall is of great practical importance in the field of biomechanics. Much attention has been paid to the propulsive mechanism of fishes and bacteria in the field of biophysics. Gray [1] studied the drag on the swimming dolphin and found that this drag is much less than that on a solid body immersed in a fluid. Gray proposed a number of mechanism which can reduce the drag, such as the effect of body shape(laminar aerofoil theory), the effect of flexible skin and the unsteady motive effect. The last one related to the fluid mechanical developments concerning swimming of fishes and has raised a question, how an unsteady movement of a body immersed in a fluid can induce a steady flow around it. A motive power of fishes is mainly due to flapping of tail and fin, a waving motion of a body has an effect of thrusting the body, and this effect reduced the drag.

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Peer review under responsibility of Egyptian Mathematical Society.

Production and hosting by Elsevier

1110-256X
http://dx.doi.org/10.1016/j.joems.2013.05.003

The problems of the flow of fluids induced by the sinusoidal wavy motion of a wall have been discussed by Burns and Parkes [2], Tanaka [3], Taylor [4] and Dhar and Nanda [5]. Tanaka studied problem both for small and moderately large Reynolds numbers. While discussing the problem for moderately large Reynolds numbers, he has shown that, if the thickness of the boundary layer is larger than the wave amplitude the technique employed for small Reynolds numbers can be applied to the case of moderately large Reynolds numbers also.

The phenomenon of peristaltic transport has enjoyed increased interest from investigators in several engineering disciplines. From a mechanical point of view peristalsis offers the opportunity of constructing pumps in which the transported medium does not come in direct contact with any moving parts such as valves, plungers and rotors. The mechanism of peristaltic transport has been also exploited for industrial applications such as sanitary fluid transport, blood pumps in heart lung mechanics and transport of corrosive fluids where the contact of the fluid with the machinery parts is prohibited. To understand peristaltic action in various situations, several theoretical and experimental investigations have been made. Important contributions to the topic on Newtonian fluid include the studies of Fung and Yih [6], Mekheimer [7], Mekheimer [8], Hayat et al. [9], Mekheimer and Abd elwahab [10], Mekheimer et al. [11], Abd elmaboud and Mekheimer [12], Srivastava and Saxena [13], Abd elmaboud et al. [14], Siddiqui and Schwarz [15], Hakeem et al. [16], etc.

Flow through a porous medium attracted the attention of many researchers in the last few decades because of its very important practical applications. It occurs in filtration of fluids and seepage of water in river beds, sandstone, limestone, bile duct, wood, the human lung, gall bladder with stones and in small blood vessels. From these studies, which discussed this point Mekheimer [17], Mekheimer and Abd elmaboud [18], Vajravelu et al. [19], Srinvas and Kothandapani [20], Ashgar et al. [21], Afsar Khan et al. [22], Khan et al. [23] and Afsar Khan et al. [24].

In several flow problems, the authors assumed adherence, i.e. that the fluid layer next to a rigid surface moves with that surface. Some authors, considered hypotheses involving slippage, i.e. a relative motion of the rigid surface and the fluid next to it. For several fluids including water and mercury, many experiments, some of them beautifully conceived and carefully performed, have indicated that the adherence condition is appropriate even when the fluid does not wet the boundary surface. From time to time, an apparently carefully experiment has seemed to lead to the opposite conclusion but further analysis has revealed theoretical or experimental error. In many applications the flow pattern corresponds to a slip flow, the fluid presents a loss of adhesion at the wetted wall making the fluid slide along the wall. In the study of fluid-solid surface interactions the concept of slip of a fluid at a solid wall serves to describe macroscopic effects of certain molecular phenomena. In all study on peristaltic flow, much works are studied with no slip condition, the effects of slip conditions discussed by Ebaid [25], Rajeev and Jain [27] and Ali et al. [26].

In all these analysis, the effects of Hall current are not considered. However, in an ionized gas, when the strength of the magnetic field is very strong, one cannot neglect the Hall effects. Attia [28] had examined unsteady Hartmann flow with heat transfer of a viscoelastic fluid taking the Hall effect into account. Hayat et al. [29] studied the Hall effects on peristaltic flow of a Maxwell fluid in a porous medium. Abo-Eldahab et al. [30], [31] investigated the effects of Hall and ion-slip currents on magnetohydrodynamic peristaltic transport and couple stress fluid.

Hence it's important to study the slip effect with Hall current on the flow induced by sinusoidal peristaltic wavy wall through a porous medium. Solutions are obtained in terms of series expansion with respect to the small amplitude by regular perturbation method. The inner (boundary layer flow) and the outer (flow beyond the boundary layer), solutions are matched by a matching process given by Kevorkian and Cole [32]. Graphs of the velocity components, both for the outer and the inner flows for various values of the problem parameters are drawn.

2. Equations of motion

We consider a two-dimensional flow of an incompressible viscous fluid due to an infinite sinusoidal wavy stretching wall of amplitude a and wave length λ, which is oscillating vertically with a frequency $\frac{c}{2\pi}$, x being the coordinate in the downstream direction of the flow and y, the coordinate perpendicular to it. The motion of the wall is described by:

$$y = h(x,t) = a\cos\frac{2\pi}{\lambda}(x - ct) \tag{1}$$

where a is the amplitude of the wavy wall, λ is the wave length, and c is the wave speed. The fundamental equations governing this model together with the generalized Ohm's law taking the effects of Hall currents and Maxwell's equations into account are:

$$\nabla.q = 0$$

$$\rho\left(\frac{\partial q}{\partial t} + (q.\nabla)q\right) = -\nabla p + \mu\nabla^2 q - \frac{\mu}{k_1}q + J \times B \tag{2}$$

$$J = \sigma\left[V \times B - \frac{1}{e\ n_e}J \times B\right] \tag{3}$$

where q is the velocity vector, p is the pressure, μ is the dynamic viscosity, $\$^2$ is the Laplacian operator, ρ is the density of the fluid, $\frac{d}{dt}$ is the material derivative, t is the time, J is the current density, B is the total magnetic field, σ is the electric conductivity, e is the electric charge, n_e is the number density of electrons. Here we assume that $\frac{a}{\lambda} \ll 1$.

The equations governing the two dimensional motion of this model are:

$$\begin{aligned}\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) &= -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{\mu}{k_1}u + \frac{\sigma B_0^2}{1+m^2}(mv - u)\\ \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) &= -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \frac{\mu}{k_1}v - \frac{\sigma B_0^2}{1+m^2}(mu + v)\end{aligned} \tag{4}$$

The boundary conditions are

$$\begin{aligned}&u = -\alpha\ \frac{\partial u}{\partial y}, \quad v = \frac{\partial h}{\partial t}\ at\ y = h(x,t)\\ &|u|, \quad |v| < \infty\ as\ y \to \infty,\end{aligned} \tag{5}$$

where u, v are the velocity components, $m = \frac{\sigma B_0}{e\ n_e}$ is the Hall parameter, and α is slip parameter.

We normalize all lengths by characteristic length $\frac{\lambda}{2\pi}$, all velocities q by characteristic speed c, the fluid pressure p by ρc^2, and the time by characteristic time $\frac{\lambda}{2\pi c}$

The above equations of motion of the fluid become,

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{R}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{u}{R\,k}+\frac{M}{R(1+m^2)}(mv-u)$$
$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\frac{1}{R}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{v}{R\,k}-\frac{M}{R(1+m^2)}(mu+v) \quad (6)$$

where Reynolds number $R=\frac{\lambda\rho c}{2\pi\mu}$, the magnetic parameter$M=\frac{\sigma B_0^2\lambda^2}{4\pi^2\mu}$, the porosity parameter $k=\frac{4\pi^2 k_1}{\lambda^2}$, and the boundary conditions are

$$u=-\beta\,\varepsilon\,\frac{\partial u}{\partial y},\quad v=\frac{\partial h}{\partial t}\ at\ y=h(x,t)$$
$$|u|,\ |v|<\infty\ as\ y\to\infty, \quad (7)$$

Where $h(x,t)=\varepsilon\cos(x-t)$, $\beta=\frac{\alpha}{a}$ and $\varepsilon=\frac{2\pi a}{\lambda}\ll 1$.

By introducing the stream function $\psi(x,\ y,\ t)$ for the fluid, the governing Eq. (6), and the boundary condition (7) become,

$$\frac{\partial}{\partial t}\nabla^2\psi+\frac{\partial\psi}{\partial x}\nabla^2\frac{\partial\psi}{\partial y}-\frac{\partial\psi}{\partial y}\nabla^2\frac{\partial\psi}{\partial x}=\frac{1}{R}\nabla^2(\nabla^2\psi)-\frac{1}{k\ R}\nabla^2-\frac{M}{R(1+m^2)}\nabla^2\psi$$
$$\frac{\partial\psi}{\partial y}=-\beta\,\varepsilon\,\frac{\partial^2\psi}{\partial y^2},\quad -\frac{\partial\psi}{\partial x}=\frac{\partial h}{\partial t}\ at\ y=h(x,t)$$
$$\left|\frac{\partial\psi}{\partial y}\right|,\left|\frac{\partial\psi}{\partial x}\right|<\infty\ as\ y\to\infty, \quad (8)$$

3. Solution of the problem

When Reynolds number becomes large, the boundary layer is formed. As we have assumed that the thickness of the boundary layer is larger than the wave amplitude, following Tanaka [5], regular perturbation technique can be applied to the present problem. If $\delta$is the thickness of the boundary layer, the non-dimensional may be defined as $\bar{y}=\frac{y}{\delta}$ and $\bar{\psi}=\frac{\psi}{\delta}$. When the viscous term is supposed to be of the same order as the inertia terms, we have that $\delta^2 R$ is 0(1) as usual. The boundary conditions at $y=h$ are expanded into Taylor series around $h=0$ in terms of the inner variables $\bar{\psi}$ and $\bar{y}$ as

$$\frac{\partial\bar{\psi}}{\partial x}(0)+\frac{h}{\delta}\frac{\partial^2\bar{\psi}}{\partial x\partial\bar{y}}(0)+\frac{1}{2}\frac{h^2}{\delta^2}\frac{\partial^3\bar{\psi}}{\partial x\partial\bar{y}^2}(0)+\cdots=-\frac{1}{\delta}\frac{\partial h}{\partial t}$$
$$\frac{\partial\bar{\psi}}{\partial\bar{y}}(0)+\frac{h}{\delta}\frac{\partial^2\bar{\psi}}{\partial\bar{y}^2}(0)+\frac{1}{2}\frac{h^2}{\delta^2}\frac{\partial^3\bar{\psi}}{\partial\bar{y}^3}(0)+\cdots=\beta\,\varepsilon\left(\frac{1}{\delta}\frac{\partial^2\bar{\psi}}{\partial\bar{y}^2}(0)+\frac{h}{\delta^2}\frac{\partial^3\bar{\psi}}{\partial\bar{y}^3}(0)+\frac{h^2}{2\,\delta^3}\frac{\partial^4\bar{\psi}}{\partial\bar{y}^4}(0)+\cdots\right) \quad (9)$$

In order that Taylor series converges, $0(\delta)$ must be larger than $0(h)$, that is, $0(\varepsilon)<0(\delta)$. Following Tanaka [5], we take $\delta=r\varepsilon^{\frac{1}{2}}$, r being an arbitrary constant of 0(1). The outer flow (the flow beyond the boundary layer) is described by (8) in terms of the original variables (ψ,x,y,t) while the inner flow (boundary layer flow) is described in terms of the inner variables $(\bar{\psi},x,\bar{y},t)$ on substituting $R=(r^2\varepsilon)^{-1}$ and $\delta=r\varepsilon^{\frac{1}{2}}$. As ε 1, we can use perturbation method and assume that (outer flow) and (inner flow) can be expanded as power series in $\varepsilon^{\frac{1}{2}}$ using,

$$\psi=\sum_{n=1}^{\infty}\varepsilon^{\frac{n}{2}}\psi_n,\ \bar{\psi}=\sum_{n=1}^{\infty}\varepsilon^{\frac{n}{2}}\bar{\psi}_n \quad (10)$$

Substituting (10) and using $\bar{y}=\left(\frac{y}{\delta}\right)$, $\bar{\psi}=\left(\frac{\psi}{\delta}\right)$, $R=(r^2\varepsilon)$, $\delta=r\varepsilon^{\frac{1}{2}}$ in (8), and the boundary conditions (9) and then equating the coefficients of like power of $\varepsilon^{\frac{1}{2}}$, We obtain the equation and the boundary conditions corresponding to first order, second order as follows.

First order$\left(\left[0\left(\varepsilon^{\frac{1}{2}}\right)\right]\right)$

OUTER

$$\frac{\partial}{\partial t}\nabla^2\psi_1=0 \quad (11)$$

INNER

$$\frac{\partial^4\bar{\psi}_1}{\partial\bar{y}^4}-\frac{\partial^3\bar{\psi}_1}{\partial t\partial\bar{y}^2}=0 \quad (12)$$

$$\frac{\partial\bar{\psi}_1}{\partial\bar{y}}(0)=0,\quad \frac{\partial\bar{\psi}_1}{\partial x}(0)=-\frac{\sin(x-t)}{r} \quad (13)$$

Second order([0(ε)])

OUTER

$$\frac{\partial}{\partial t}\nabla^2\psi_2=\frac{\partial\psi_1}{\partial x}\nabla^2\frac{\partial\psi_1}{\partial y}-\frac{\partial\psi_1}{\partial y}\nabla^2\frac{\partial\psi_1}{\partial x} \quad (14)$$

INNER

$$\frac{\partial^4\bar{\psi}_2}{\partial\bar{y}^4}-\frac{\partial^3\bar{\psi}_2}{\partial t\partial\bar{y}^2}=\frac{\partial\bar{\psi}_1}{\partial\bar{y}}\frac{\partial^3\bar{\psi}_1}{\partial\bar{y}^2\partial x}-\frac{\partial\bar{\psi}_1}{\partial\bar{x}}\frac{\partial^3\bar{\psi}_1}{\partial\bar{y}^3} \quad (15)$$

$$\frac{\partial\bar{\psi}_2}{\partial\bar{y}}(0)=-\frac{\beta}{r}\frac{\partial^2\bar{\psi}_1}{\partial\bar{y}^2}(0)-\frac{\cos(x-t)}{r}\frac{\partial^2\bar{\psi}_1}{\partial\bar{y}^2}(0)$$
$$\frac{\partial\bar{\psi}_2}{\partial\bar{x}}(0)=-\frac{\cos(x-t)}{r}\frac{\partial^2\bar{\psi}_1}{\partial x\partial\bar{y}}(0) \quad (16)$$

Third order$\left(\left[0\left(\varepsilon^{\frac{3}{2}}\right)\right]\right)$

OUTER

$$\frac{\partial}{\partial t}\nabla^2\psi_3=r^2\nabla^2\nabla^2\psi_1+\frac{r^2}{k}\nabla^2\psi_1+\frac{r^2M}{1+m^2}\nabla^2\psi_1-\frac{\partial\psi_1}{\partial y}\nabla^2\frac{\partial\psi_2}{\partial x}-\frac{\partial\psi_2}{\partial y}\nabla^2\frac{\partial\psi_1}{\partial x}+\frac{\partial\psi_1}{\partial x}\nabla^2\frac{\partial\psi_2}{\partial y}+\frac{\partial\psi_2}{\partial x}\nabla^2\frac{\partial\psi_1}{\partial y} \quad (17)$$

INNER

$$\frac{\partial^4\bar{\psi}_3}{\partial\bar{y}^4}-\frac{\partial^3\bar{\psi}_3}{\partial t\partial\bar{y}^2}=-2r^2\frac{\partial^4\bar{\psi}_1}{\partial^2 x\partial^2\bar{y}}+r^2\frac{\partial^3\bar{\psi}_1}{\partial t\partial x^2}+\frac{r^2}{k}\frac{\partial^2\bar{\psi}_1}{\partial\bar{y}^2}+\frac{r^2M}{1+m^2}\frac{\partial^2\bar{\psi}_1}{\partial\bar{y}^2}+\frac{\partial\bar{\psi}_1}{\partial\bar{y}}\frac{\partial^3\bar{\psi}_2}{\partial\bar{y}^2\partial x}+\frac{\partial\bar{\psi}_2}{\partial\bar{y}}\times\frac{\partial^3\bar{\psi}_1}{\partial\bar{y}^2\partial x}-\frac{\partial\bar{\psi}_1}{\partial\bar{x}}\frac{\partial^3\bar{\psi}_2}{\partial\bar{y}^3}-\frac{\partial\bar{\psi}_2}{\partial\bar{x}}\frac{\partial^3\bar{\psi}_1}{\partial\bar{y}^3} \quad (18)$$

$$\frac{\partial\bar{\psi}_3}{\partial\bar{y}}(0)=-\frac{\beta}{r}\frac{\partial^2\bar{\psi}_2}{\partial\bar{y}^2}(0)-\frac{\beta}{r^2}\cos(x-t)\frac{\partial^3\bar{\psi}_1}{\partial\bar{y}^3}(0)-\frac{1}{r}\times\cos(x-t)\frac{\partial^2\bar{\psi}_2}{\partial\bar{y}^2}(0)-\frac{1}{2r^2}\cos^2(x-t)\frac{\partial^3\bar{\psi}_1}{\partial\bar{y}^3}\times(0)\frac{\partial\bar{\psi}_3}{\partial\bar{x}}(0)$$
$$=-\frac{1}{r}\cos(x-t)\frac{\partial^2\bar{\psi}_2}{\partial x\partial\bar{y}}(0)-\frac{1}{2r^2}\cos^2(x-t)\times\frac{\partial^3\bar{\psi}_1}{\partial x\partial\bar{y}^2}(0) \quad (19)$$

Fourth order($[0(\varepsilon^2)]$)

OUTER

$$\frac{\partial}{\partial t}\nabla^2\psi_4 = r^2\nabla^2\nabla^2\psi_2 - \frac{r^2}{k}\nabla^2\psi_2 - \frac{r^2 M}{1+m^2}\nabla^2\psi_2 - \frac{\partial\psi_1}{\partial y}\nabla^2\frac{\partial\psi_3}{\partial x} - \frac{\partial\psi_2}{\partial y}\nabla^2\frac{\partial\psi_2}{\partial x} - \frac{\partial\psi_3}{\partial y}\nabla^2\frac{\partial\psi_1}{\partial x} + \frac{\partial\psi_1}{\partial x}\nabla^2\frac{\partial\psi_3}{\partial y} + \frac{\partial\psi_2}{\partial x}\nabla^2\frac{\partial\psi_2}{\partial y} + \frac{\partial\psi_3}{\partial x}\nabla^2\frac{\partial\psi_1}{\partial y} \quad (20)$$

INNER

$$\frac{\partial^4\bar\psi_4}{\partial\bar y^4} - \frac{\partial^3\bar\psi_4}{\partial t\partial\bar y^2} = -2r^2\frac{\partial^4\bar\psi_2}{\partial^2 x\partial^2\bar y} + r^2\frac{\partial^3\bar\psi_2}{\partial t\partial x^2} + \frac{r^2}{k}\frac{\partial^2\bar\psi_2}{\partial\bar y^2} + \frac{r^2 M}{1+m^2}\frac{\partial^2\bar\psi_2}{\partial\bar y^2} + r^2\frac{\partial\bar\psi_1}{\partial\bar y}\frac{\partial^3\bar\psi_1}{\partial x^3} - r^2\frac{\partial\bar\psi_1}{\partial\bar x} \times \frac{\partial^3\bar\psi_1}{\partial x^2\partial y} + \frac{\partial\bar\psi_1}{\partial\bar y}\frac{\partial^3\bar\psi_3}{\partial\bar y^2\partial x} + \frac{\partial\bar\psi_2}{\partial\bar y}\frac{\partial^3\bar\psi_2}{\partial\bar y^2\partial x} + \frac{\partial\bar\psi_3}{\partial\bar y}\frac{\partial^3\bar\psi_1}{\partial\bar y^2\partial x} - \frac{\partial\bar\psi_1}{\partial x}\frac{\partial^3\bar\psi_3}{\partial\bar y^3} - \frac{\partial\bar\psi_2}{\partial x}\frac{\partial^3\bar\psi_2}{\partial\bar y^3} - \frac{\partial\bar\psi_3}{\partial x}\frac{\partial^3\bar\psi_1}{\partial\bar y^3} \quad (21)$$

$$\frac{\partial\bar\psi_4}{\partial\bar y}(0) = -\frac{\beta}{r}\frac{\partial^2\bar\psi_3}{\partial\bar y^2}(0) - \frac{\beta}{r^2}\cos(x-t)\frac{\partial^3\bar\psi_2}{\partial\bar y^3}(0) - \frac{\beta}{2r^3}\cos^2(x-t)\frac{\partial^4\bar\psi_1}{\partial\bar y^4}(0) - \frac{1}{r}\cos(x-t)\frac{\partial^3\bar\psi_3}{\partial\bar y^2}(0) - \frac{1}{2r^2}\cos^2(x-t)\frac{\partial^3\bar\psi_2}{\partial\bar y^3}(0) - \frac{1}{6r^3}\cos^3(x-t)\frac{\partial^4\bar\psi_1}{\partial\bar y^4}(0)$$

$$\frac{\partial\bar\psi_4}{\partial\bar x}(0) = -\frac{1}{r}\cos(x-t)\frac{\partial^2\bar\psi_3}{\partial x\partial\bar y}(0) - \frac{1}{2r^2}\cos^2(x-t)\frac{\partial^3\bar\psi_2}{\partial x\partial\bar y^2}(0) - \frac{1}{6r^3}\cos^3(x-t)\frac{\partial^4\bar\psi_1}{\partial\bar y^3\partial x}(0) \quad (22)$$

A series of the inner solutions should satisfy the boundary conditions on the wall,while the outer solutions are only restricted to be bounded as y increases, but is

$$\left|\frac{\partial\psi_n}{\partial x}\right|, \left|\frac{\partial\psi_n}{\partial y}\right| < \infty \ as\ y\to\infty\ for\ n = 1,2,3,\ldots.$$

It is necessary to match the outer and the inner solutions, Following cole xxxx the matching is carried out for both x and y components f the velocity by the following principles:

$$\lim_{\varepsilon\to 0}\frac{1}{\varepsilon^{\frac{N}{2}}}\left[\sum_{n=1}^{N}\varepsilon^{\frac{n}{2}}\frac{\partial\psi_n}{\partial y} - \sum_{n=1}^{N}\varepsilon^{\frac{n}{2}}\frac{\partial\bar\psi_n}{\partial\bar y}\right] = 0 \quad (23)$$

$$\lim_{\varepsilon\to 0}\frac{1}{\varepsilon^{\frac{N}{2}}}\left[\sum_{n=1}^{N}\varepsilon^{\frac{n}{2}}\frac{\partial\psi_n}{\partial x} - r\varepsilon^{\frac12}\sum_{n=1}^{N}\varepsilon^{\frac{n}{2}}\frac{\partial\bar\psi_n}{\partial x}\right] = 0 \quad (24)$$

where $\bar y$ is fixed up to Nth order of magnitude, let us find out first order solutions in the form:

$$\bar\psi_1(x,\bar y,t) = F_1(\bar y)e^{i(x-t)} + F_1^*(\bar y)e^{-i(x-t)} + F_{1s}(\bar y)$$
$$\psi_1(x,y,t) = f_1(y)e^{i(x-t)} + f_1^* e^{-i(x-t)} + f_{1s}(y) \quad (25)$$

By substituting (25) in the first order differential equations (11) and (12) and the boundary conditions (13) we obtain the following system of equations

$$\frac{d^4F_1}{d\bar y^4} - i\frac{d^2F_1}{d\bar y^2} = 0,\quad \frac{d^4F_{1s}}{d\bar y^4} = 0,\quad \frac{d^2 f_1}{dy^2} - f_1 = 0,\quad \frac{d^4F_{1s}}{d\bar y^4} = 0 \quad (26)$$

and their solutions

$$F_1 = A_1 e^{-\lambda\bar y} + \lambda A_1\bar y - A_1 + \frac{1}{2r}$$
$$\frac{dF_{1s}}{d\bar y} = B_1\bar y^2 + B_2\bar y$$
$$f_1 = ae^{-y} \quad (27)$$
$$Following\ tanaka[3]\ \frac{df_{1s}}{dy} = C_1$$

where $\lambda = \sqrt{-i}$ and A, B, a are constants. Substituting (27) into (23), we have

$$\lim_{\varepsilon\to 0}\frac{1}{\varepsilon^{\frac12}}\left[\varepsilon^{\frac12}\frac{\partial\psi_1}{\partial y} - \varepsilon^{\frac12}\frac{\partial\bar\psi_1}{\partial\bar y}\right] = \lim_{\varepsilon\to 0}[-ae^{-y}e^{i(x-t)} + c.c. + C_1 - (-A_1\lambda e^{-\lambda\bar y} + \lambda A_1 + c.c.) - B_1\bar y^2 - B_2\bar y] = 0$$

where $c.c.$ stands for the corresponding complex conjugate. Taking account that $y = r\varepsilon^{\frac12}\bar y$, expanding the exponential as

$$e^{-y} = e^{-r\varepsilon^{\frac12}\bar y} = 1 - r\varepsilon^{\frac12}\bar y + r^2\varepsilon\bar y^2 + \cdots$$

and noting that $\exp(-\lambda\bar y) = \exp\left(\frac{-\lambda}{r\varepsilon^{\frac12}}y\right)$ decays very rapidly as $\varepsilon\to 0$ (which is called transcendentally small (T.S.T) and is neglected in the matching process), we have

$$\lim_{\varepsilon\to 0}[(-a-\lambda A_1) + c.c. + C_1 - B_1\bar y^2 - B_2\bar y + T.S.T] = 0$$

Thus the matching condition is satisfied only if

$$-a-\lambda A_1,\ B_1 = B_2 = 0,\ C_1 = 0$$

when similar process is carried out for (24), we have

$$\lim_{\varepsilon\to 0}\frac{1}{\varepsilon^{\frac12}}\left[\varepsilon^{\frac12}\frac{\partial\psi_1}{\partial x} - r\varepsilon^{\frac12}\frac{\partial\bar\psi_1}{\partial x}\right] = \lim_{\varepsilon\to 0}\frac{1}{\varepsilon^{\frac12}}\left[\varepsilon^{\frac12}(ia)e^{-y}e^{i(x-t)} + c.c. + o(\varepsilon)\right] = 0$$

so that matching condition is satisfied $a = 0$. Thus we have

$$A_1 = B_1 = B_2 = 0$$
$$C_1 = 0$$

and the first order solution are obtained as

$$\psi_1 = 0$$
$$\bar\psi_1 = \frac{1}{2r}e^{i(x-t)} + \frac{1}{2r}e^{-i(x-t)} \quad (28)$$

Next we seek the solutions $\psi_2, \bar\psi_2$ in the following form

$$\bar\psi_2(x,\bar y,t) = F_2(\bar y)e^{2i(x-t)} + F_{21}(\bar y)e^{i(x-t)} + c.c. + F_{2s}(\bar y)$$
$$\psi_2(x,y,t) = f_1(y)e^{2i(x-t)} + f_{21}e^{i(x-t)} + c.c. + f_{2s}(y) \quad (29)$$

Substituting (29)and (28) into (14)–(16) we get after some calculations

$$\bar\psi_2 = \left(-\frac{i\lambda}{2}e^{-\lambda\bar y} - \frac{\bar y}{2} + \frac{i\lambda}{2}\right)e^{i(x-t)} + c.c.$$
$$\psi_2 = \frac{1}{2}e^{-y}e^{i(x-t)} + c.c. \quad (30)$$

Let us now seek third order solutions in the form

$$\bar\psi_3(x,\bar y,t) = F_3(\bar y)e^{3i(x-t)} + F_{32}(\bar y)e^{2i(x-t)} + F_{31}(\bar y)e^{i(x-t)} + c.c. + F_{3s}(\bar y)$$
$$\psi_3(x,y,t) = f_3(y)e^{3i(x-t)} + f_{31}e^{2i(x-t)} + f_{32}(y)e^{i(x-t)} + c.c. + f_{3s}(y) \quad (31)$$

where

$$\begin{aligned}
&F_3 = 0\\
&F_{32} = \frac{1}{4r} - \frac{1}{4r}e^{-\lambda\bar{y}}\\
&F_{31} = \left(-\frac{ir}{2} - \frac{\beta}{2r}\right)e^{-\lambda\bar{y}} - \frac{i\lambda r\bar{y}}{2} + \frac{r\bar{y}^2}{4} + \frac{ir}{2} + \frac{\beta}{2r}\\
&f_{31} = \frac{ir\lambda}{2}e^{-y}\\
&\frac{dF_{3S}}{d\bar{y}} = \frac{\lambda}{4r}e^{-\lambda\bar{y}} + \frac{\lambda^*}{4r}e^{-\lambda^*\bar{y}}
\end{aligned} \tag{32}$$

We shall now seek the fourth order solutions in the following form

$$\begin{aligned}
&\bar{\psi}_4(x,\bar{y},t) = F_4(\bar{y})e^{4i(x-t)} + F_{43}(\bar{y})e^{3i(x-t)} + F_{42}(\bar{y})e^{2i(x-t)}\\
&\quad + F_{41}(\bar{y})e^{i(x-t)} + c.c. + F_{3s}(\bar{y})\\
&\psi_4(x,y,t) = f_4(y)e^{4i(x-t)} + f_{43}e^{3i(x-t)} + f_{42}(y)e^{2i(x-t)}\\
&\quad + f_{41}(y)e^{i(x-t)} + c.c. + f_{3s}(y)
\end{aligned} \tag{33}$$

where

$$\begin{aligned}
&F_4 = 0\\
&F_{43} = \frac{-\lambda}{16r^2}e^{-\lambda\bar{y}}\\
&F_{42} = \left(\frac{\bar{y}}{4} - \frac{\beta\,\lambda}{4r^2} - \frac{i\lambda}{4}\right)e^{-\lambda\bar{y}} - \frac{\bar{y}}{2} - \frac{\beta\,\lambda}{8r^2} + \frac{i\lambda}{4}\\
&F_{41} = \left(-\frac{\lambda}{8r^2} + \frac{r^2\lambda}{4} - \frac{i\lambda\beta}{2} - \frac{\lambda\beta^2}{2r^2} - \frac{\lambda r^2 M}{4(1+m^2)} - \frac{\lambda r^2}{4k}\right)\\
&\quad e^{-\lambda\bar{y}} + \left(\frac{ir^2\bar{y}}{4} + \frac{ir^2M\bar{y}}{4(1+m^2)} + \frac{ir^2\bar{y}}{4k}\right)e^{-\lambda\bar{y}} - \frac{\lambda^*}{16r^2}e^{-\lambda^*\bar{y}} - \frac{r^2\bar{y}^3}{12}\\
&\quad + \frac{ir^2\lambda\bar{y}^2}{4} - \frac{ir^2\bar{y}}{2} - \frac{\beta\bar{y}}{2} - \frac{\lambda r^2}{4} + \frac{i\beta\lambda}{2} + \frac{\lambda\beta^2}{2r^2} + \frac{\lambda r^2 M}{4(1+m^2)} + \frac{\lambda r^2}{4k}
\end{aligned} \tag{34}$$

$$\begin{aligned}
\frac{dF_{4S}}{d\bar{y}} = &\frac{1}{4}e^{-(\lambda+\lambda^*)\bar{y}} - \left(\frac{1}{4} + \frac{i\beta}{4r^2} + \frac{\lambda\bar{y}}{4} + \frac{i}{4}\right)e^{-\lambda\bar{y}}\\
&- \left(\frac{1}{4} - \frac{i\beta}{4r^2} + \frac{\lambda^*\bar{y}}{4} - \frac{i}{4}\right)e^{-\lambda^*\bar{y}} + \frac{1}{4}
\end{aligned}$$

$$f_4 = f_{43} = 0, \quad f_{42} = \frac{1}{4}e^{-2y}, \quad f_{41} = \left(\frac{ir^2}{2} + \frac{\beta}{2}\right)e^{-y}, \quad \frac{df_{4s}}{dy} = \frac{1}{4}$$

4. Discussion problem

Here, we should note that the third order solution has a steady streaming component F_{3s}. However, it attenuates very rapidly as $\bar{y}$ increases and is confined only in the boundary layer, while no steady streaming is induced in the outer layer up to this order of approximation. But the fourth order solutions consist of

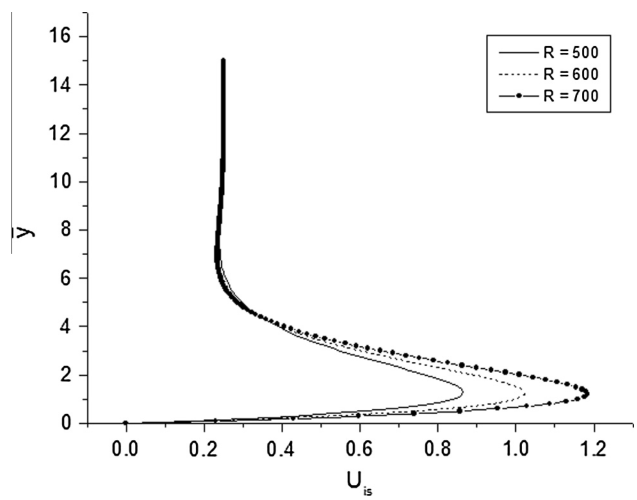

Figure 2 Induced steady axial velocity component of the fluid U_{is} in the boundary layer for different values of the Reynolds number R at $\beta = 0.1$ and $\varepsilon = 0.1$.

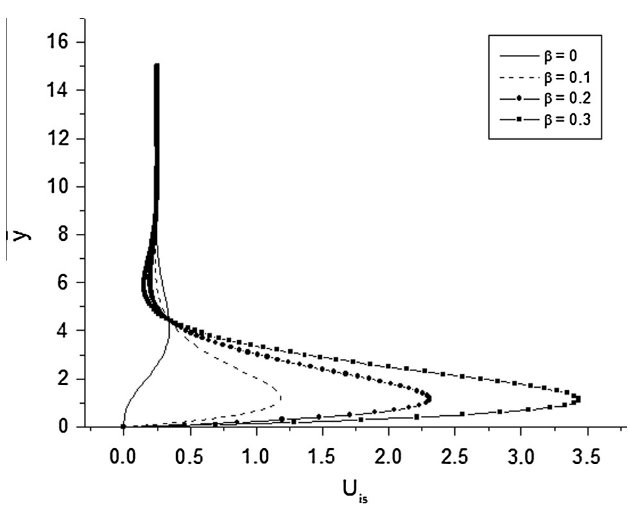

Figure 1 Induced steady axial velocity component of the fluid U_{is} in the boundary layer for different values of the slip parameter β at $R = 500$ and $\varepsilon = 0.1$.

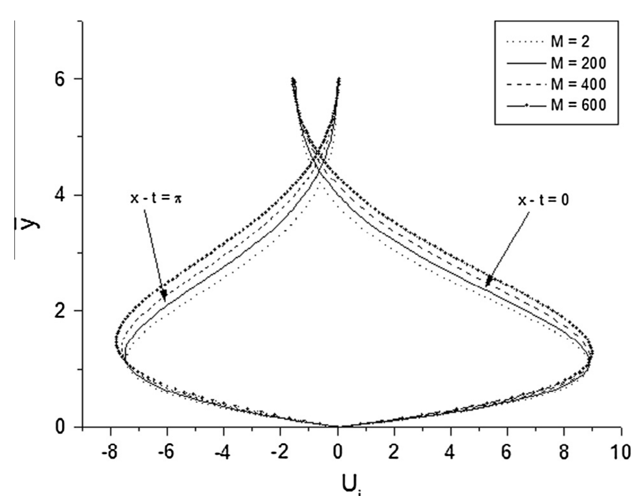

Figure 3 Axial velocity component of the fluid U_i in the boundary layer for different values of the magnetic parameter M at $R = 500$, $\varepsilon = 0.1$, $\beta = 0.1$, $k = 1$ and $m = 0.5$.

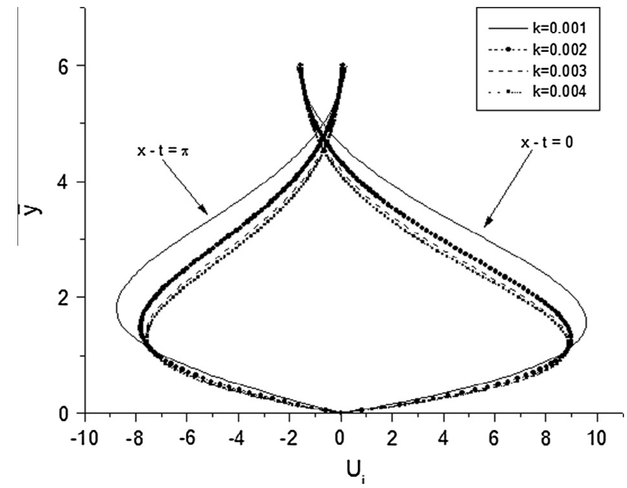

Figure 4 Axial velocity component of the fluid U_i in the boundary layer for different values of the porosity parameter k at $R = 500$, $\varepsilon = 0.1$, $M = 2$, $\beta = 0.1$ and $m = 0.5$.

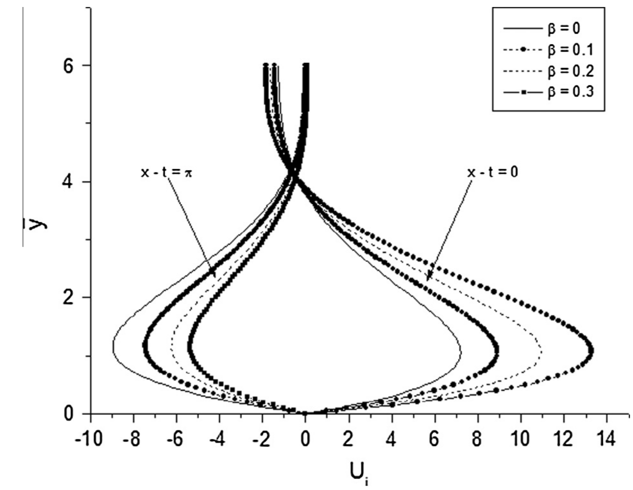

Figure 6 Axial velocity component of the fluid U_i in the boundary layer for different values of the slip parameter at $R = 500$, $\varepsilon = 0.1$, $M = 2$, $m = 0.5$ and $k = 1$.

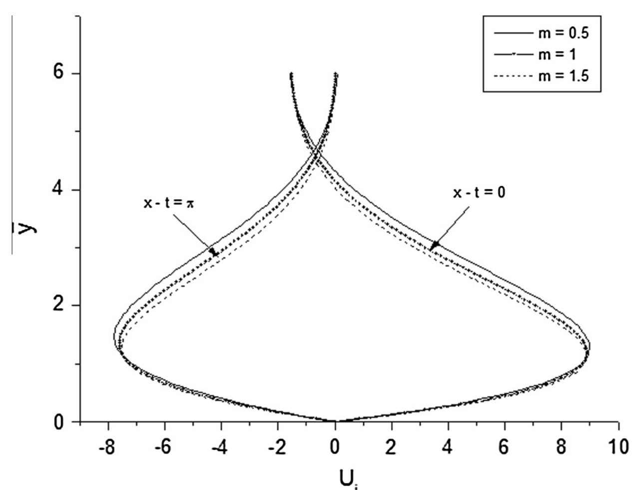

Figure 5 Axial velocity component of the fluid U_i in the boundary layer for different values of the Hall parameter m at $R = 500$, $\varepsilon = 0.1$, $M = 600$, $\beta = 0.1$ and $k = 1$.

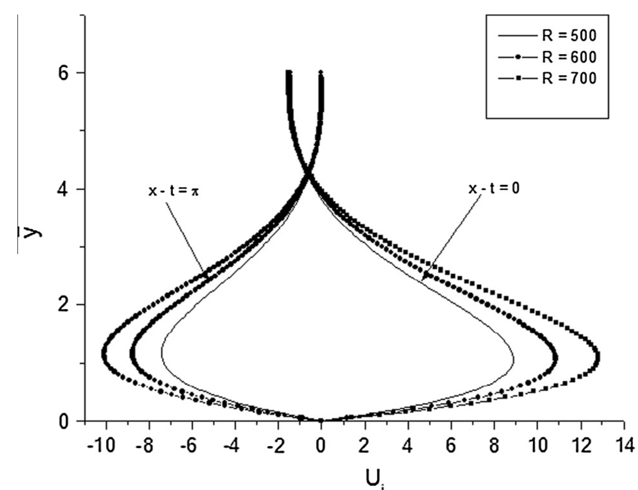

Figure 7 Axial velocity component of the fluid U_i in the boundary layer for different values of the Reynolds number R at $\varepsilon = 0.1$, $M = 2$, $\beta = 0.1$, $m = 0.5$, $k = 1$.

the steady part in addition to the periodic one, so we shall take up for discussion the fourth order solution.

We may say that the progressive motion of the wall causes, at first, the periodic flow in the boundary layer having the same phase as that of the wall motion and then it causes flows of higher harmonic in the boundary layer and induces the periodic flow in the outer layer successively. The components of velocities for outer and inner flows have been plotted against y and $\bar{y}$ respectively for various values of the Reynolds numbers$R$, the slip parameterβ, the magnetic parameter$M$, the Hall parameter m and $x - t$, taking $\varepsilon = 0.1$.

Figs. 1 and 2 illustrates the behavior of the inner steady streaming flow U_{is}. We find from Fig. 1 the fluid of the inner steady streaming part approach to a constant value in the form of the damped oscillation with respect to the distance from the wall and the increase of the slip parameter β increases the velocity of the inner steady streaming flow U_{is}. When the slip parameter $\beta = 0$, the same results that given by Tanaka [3] in the case of large Reynolds number are obtained, where the steady flow velocity approaches 1/4 away from the wall. Fig. 2 shows that the greater of the Reynolds number R the higher of the velocity of the inner steady streaming flow. It is also, should be remarked that when Reynolds number becomes very large, the steady flow velocity approaches to 1/4 away from the wall, which coincides with the results of [J Phy Soc Japan]

Now we will study the nature of the inner axial velocityU_i through the Figs. 3–7. It is clear that from Fig. 3 at $x - t = 0$, the increases of the magnetic parameter M slightly decrease the velocity of the inner axial velocity U_i in the region very close to the wall and as y increases and far away from the wall, the increase of the magnetic field accelerate the inner axial velocity U_i, and contrast in the case of $x - t = \pi$. It is also interesting to note that the inner axial velocity U_i become stea-

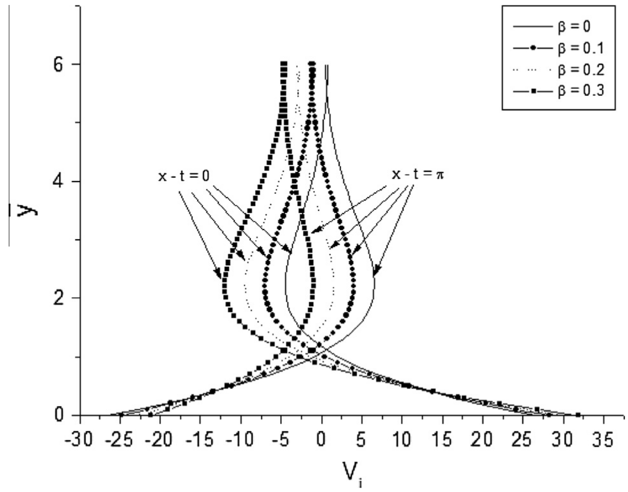

Figure 8 Transverse velocity component of the fluid V_i in the boundary layer for different values of the slip parameter β at $R = 500$, $\varepsilon = 0.1$, $M = 2$, $k = 1$ and $m = 0.5$.

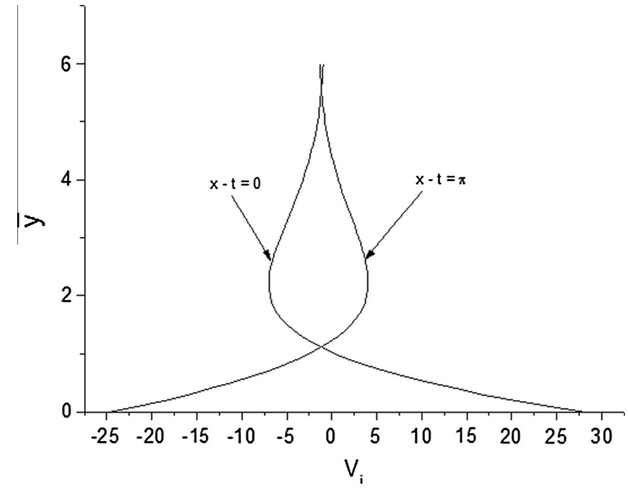

Figure 10 Transverse velocity component of the fluid V_i in the boundary layer for different values of the magnetic parameter M, at $R = 500$, $\varepsilon = 0.1$ and $\beta = 0.1$.

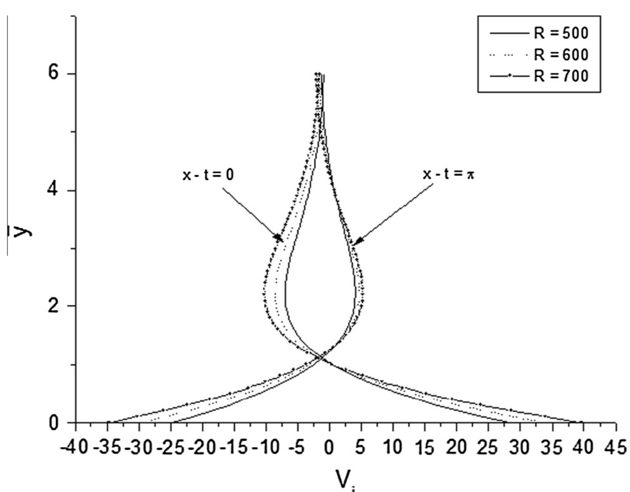

Figure 9 Transverse velocity component of the fluid V_i in the boundary layer for different values of the Reynolds number R at $\varepsilon = 0.1$, $M = 2$, $\beta = 0.1$, $m = 0.5$ and $k = 1$.

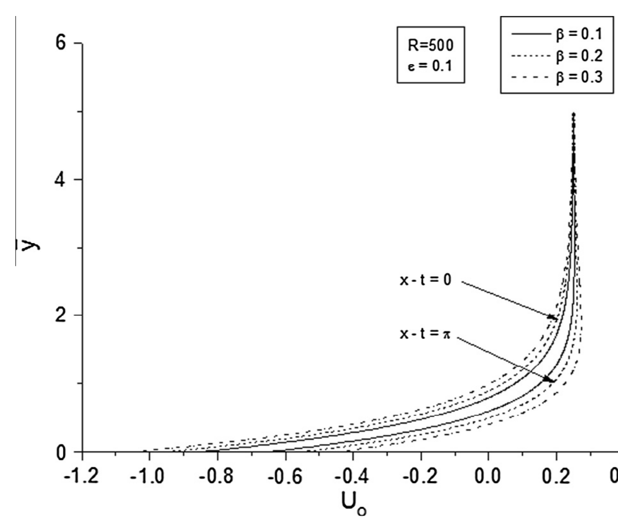

Figure 11 Outer flow axial velocity component of the fluid U_0 for different values of the slip parameter β at $\varepsilon = 0.1$, $M = 2$, $m = 0.5$ and $k = 1$.

dy as y increases and approach almost equal value. In Fig. 4 we find that the effect of the porosity parameter$K$ have a reverses influence of that of the effect of the magnetic parameter M on the inner velocity U_i. It is noted from Fig. 5 at $x - t = 0$ that the increasing of the Hall parameter m produce a slight increases in the inner axial velocity U_i near the wall whereas a reverse effect is observed far a way from the wall while the inverse behaviour for the curves occurs at $x - t = \pi$. Fig. 6 indicate that the increases of the slip parameter β increases the inner axial velocity U_i for at $x - t = 0$ and vice versa at $x - t = \pi$. Through Fig. 7 we see that the effect of the Reynolds number R increases the inner axial velocity U_i at $x - t = 0$ and vice versa at $x - t = \pi$, and it is interesting to note that the inner axial velocity U_i is oscillating between positive and negative values for all figures and steady as y increases, where we go away from the wall.

Now we will study the inner transverse velocityV_i through Figs. 8 and 9. From Fig. 8 we find that the increase of the slip parameter β decreases the inner transverse velocityV_i. And through Fig. 9 we see that the effect of increasing Reynolds number R increases the inner transverse velocity V_i at $x - t = \pi$, but the inverse occurs at $x - t = 0$. But we see that the change of the magnetic parameter M do not make an effect in the inner transverse velocity V_i and gives the results as a constant value as shown in Fig. 10. The same effect for different values of the Hall parameter m and porosity parameter k.

Fig. 11 describes the axial outer velocity U_o, and from this figure, we see that the increases of the slip parameter β increase the outer axial velocity U_o at $x - t = \pi$ and decreases at $x - t = 0$. Also we see that as y increases al curves of the outer axial velocityU_o approach almost to an equal value, and we find that the velocity U_o for $x - t = 0$ is less than that for $x - t = \pi$.

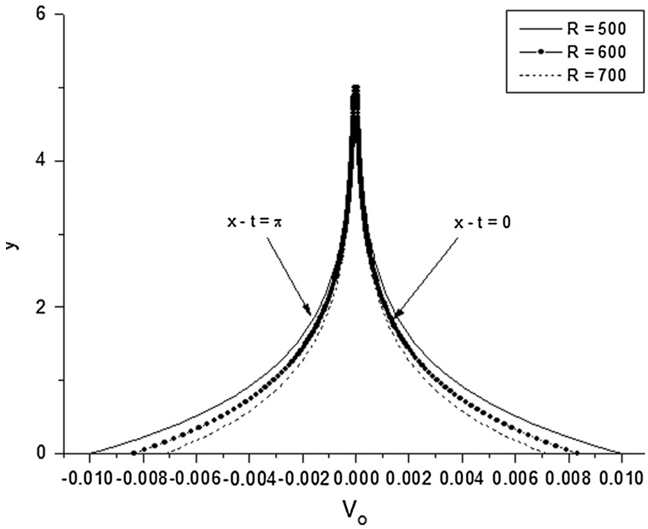

Figure 12 Outer flow transverse velocity component of the fluid V_o for different values of the Reynolds number R at $\varepsilon = 0.1$ and $\beta = 0.1$.

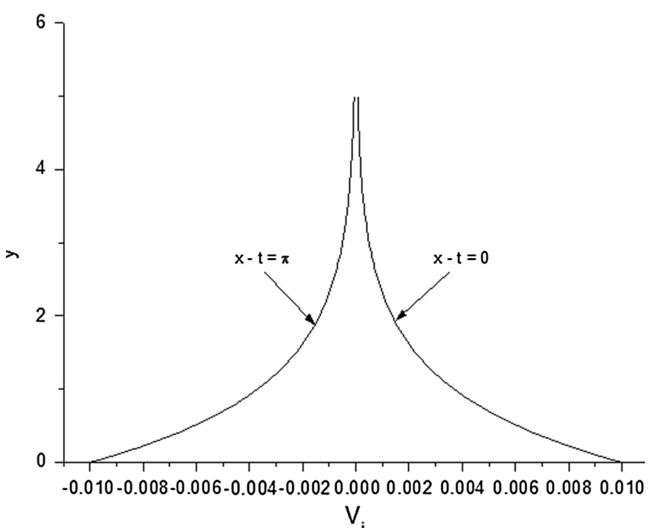

Figure 13 Outer flow transverse velocity component of the fluid V_o for different values of the slip parameter β at $\varepsilon = 0.1$ and $R = 500$.

From Fig. 12 we observe the nature of the transverse outer velocity V_o. The increases of Reynolds number R increase the transverse outer velocity V_o for $x - t = \pi$ and decrease the transverse outer velocity V_o for $x - t = 0$. But for both $x - t = \pi$ and $x - t = 0$ the transverse outer velocity V_o becomes steady as y increases and the transverse outer velocity V_o for $x - t = \pi$ is less than $x - t = 0$, and the different values of the slip parameter β do not make an effect on the transverse outer velocity V_o and give the same results as shown in Fig. 13.

Acknowledgments

Authors thank the referee for his valuable comments that have enhanced the presentation of the paper.

References

[1] J. Gray, Peristaltic motion, J. Exp. Biol. 13 (1936) 192.
[2] C. BurnsJ, T. Parkes, Peristaltic motion, J. Fluid Mech. 29 (1967) 731–734.
[3] K. Tanaka, Induced flow due to wavy motion of a wall: I. The case of small and moderately large Reynolds numbers, J. Phys. Soc. Jpn. 42 (1) (1977) 297–305.
[4] G. Taylor, Analysis of the swimming of microscopic organisms, Proc. Roy. Soc. Lond. Ser. A 209 (1951) 447–461.
[5] H. Dhar, K.D. Nanda, Induced flow due to normal oscillation of a wavy wall, Bull. Calcutta Math. Soc. 81 (5) (1989) 382–393.
[6] Y.C. Fung, C.S. Yih, Peristaltic transport, J. Appl. Mech. 35 (1968) 669–675.
[7] Kh. S. Mekheimer, Peristaltic transport of a couple stress fluid in a uniform and non-uniform channels, Biorheology 39 (2004) 755–765.
[8] Kh. S. Mekheimer, Effect of induced magnetic field on peristaltic flow of a couple stress Fluid, Phys. Lett. A 372/23 (2008) 4271–4278.
[9] T. Hayat, Kh. Yasir, Kh. S. Mekheimer, A. Nasir, Effect of an induced magnetic field on the Peristaltic flow of a third-order fluid, Numer. Methods Partial Differ. Eq. J. (NMPDE) 26 (2010) 345–366.
[10] Kh. S. Mekheimer, A.N. Abdel-Wahab, Net annulus flow of a compressible viscous liquid with peristalsis, J. Aerosp. Eng. 25 (4) (2012) 660–669.
[11] Kh.S. Mekheimer, S. Najma, T. Hayat, Simultaneous effects of induced magnetic field and heat and mass transfer on the peristaltic motion of second-order fluid in a channel, in: International Journal for Numerical Methods in Fluids 70 (2012) 342–358.
[12] Y. Abd elmaboud, Kh.S. Mekheimer, Non-linear peristaltic transport of a second-order fluid through a porous medium, Applied Mathematical Modelling 35 (2011) 2695–2710.
[13] V.P. Srivastava, M. Saxena, A two-fluid model of non-Newtonian blood flow induced by peristaltic waves, Rheol. Acta 34 (1995) 406–414.
[14] Y. Abd elmaboud, Kh.S. Mekheimer, A. Abdellatif, Thermocouple-stress fluid in an Asymmetric channel with peristalsis, Journal of Heat Transfer, ASME publications 135 (2013) 044502-1.
[15] A.M. Siddiqui, W.H. Schwarz, Peristaltic flow of a second order fluid in tubes, J. Non-Newtonian Fluid Mech. 53 (1994) 257–284.
[16] A.E. Hakeem, A.E. Naby, A.E.M.E. Misery, Effects of an endoscope and generalized Newtonian fluid on peristaltic motion, Appl. Math. Comput. 128 (2002) 19–35.
[17] Kh.S. Mekheimer, Non-linear peristaltic transport through a porous medium in an inclined planar channel, J. Porous Media 6 (3) (2003) 189–201.
[18] Kh. S. Mekheimer, Y. Abd elmaboud, Peristaltic flow through a porous medium in an annulus: application of an endoscope, Appl. Math. Inform. Sci. 2 (1) (2008) 103–111.
[19] K. Vajravelu, G. Radhakrishnamacharya, V. Radhakrishnamurty, peristaltic flow and heat transfer in a vertical porous annulus with long-wavelength approximation, Int. J. Nonlinear Mech. Math. 42 (2007) 754–759.
[20] S. Srinivas, M. Kothandapani, The influence of heat and mass transfer on MHD peristaltic flow through a porous space with complaint walls, Appl. Math. Comput. 213 (2009) 197–208.
[21] S. Ashgar, C.M. Khalique, R. Ellahi, Influence of a partial slip on flows of a second grade fluid in a porous medium, J. Porous Media 10 (8) (2007) 797–805.
[22] A. Afsar Khan, R. Ellahi, K. Vafai, Peristaltic transport of Jeffrey fluid with variable viscosity through a porous medium in an asymmetric channel, Advances in Mathematical Physics, vol.

2012, Article ID 169642, 15 pp. doi:http://dx.doi.org/10.1155/2012/169642.

[23] I. Khan, R. Ellahi, C. Fetecau, Some MHD flows of a second grade fluid through the porous medium, J. Porous Media 11 (4) (2008) 389–400.

[24] A. Afsar Khan, R. Ellahi, M. Usman, Effects of variable viscosity on the flow of non-Newtonian fluid through a porous medium in an inclined channel with slip conditions, J. Porous Media 16 (1) (2013) 59–67.

[25] A. Ebaid, Effects of magnetic field and wall slip conditions on the peristaltic transport of a Newtonian fluid in an asymmetric channel, Phys. Lett. A 372 (2008) 4493–4499.

[26] N. Ali, Q. Hussain, T. Hayat, S. Asghar, Slip effects on the peristaltic transport of MHD fluid with variable viscosity, Phys. Lett. A 372 (2008) 1477–1489.

[27] R. Taneja, N.C. Jain, MHD flow with slip effects and temperature-dependent heat source in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall, Defence Sci. J. 54 (2004) 21–29.

[28] H.A. Attia, Unsteady Hartmann flow with heat transfer of a viscoelastic fluid considering the Hall effect, Can. J. Phys. 82 (2004) 12739.

[29] T. Hayat, N. Ali, S. Asghar, Hall effects on peristaltic flow of a Maxwell fluid in a porous medium, Phys. Lett. A 363 (2007) 397–403.

[30] E.M. Abo-Eldahab, E.I. Barakat, K.I. Nowar, Hall currents and ion-slip effects on the MHD peristaltic transport, Int. J. Appl. Math. Phys. (IJAMP) 2 (2010) 113–123.

[31] E.M. Abo-Eldahab, E.I. Barakat, K.I. Nowar, Effects of Hall and ion-slip currents on peristaltic transport of a couple stress fluid, Int. J. Appl. Math. Phys. (IJAMP) 2 (2010) 145–157.

[32] J. Kevorkian, J.D. Cole, Applied Mathematical Sciences, Perturbation Methods in Applied Mathematics, vol. 34, Springer-Verlag, New York, 1981, MR 82g: 34082. Zbl 456.34001.